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LINEARIZED THEORY FOR FLOWS ABOUT LIFTING FOILS AT ZERO  
CAVITATION NUMBER

by

M.P. Tulin and M.P. Burkart

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# NOTATION

$A_0$  The Fourier coefficient of zero order  $= -\frac{1}{\pi} \int_0^\pi \frac{d\bar{y}_0}{ds}(\theta) d\theta$

$A_n$  The Fourier coefficient of nth order  $= -\frac{2}{\pi} \int_0^\pi \frac{d\bar{y}_0}{ds}(s) \cos n\theta d\theta$

$a, b$  Constants

$C_D$  Drag coefficient  $= \frac{D}{\frac{1}{2}\rho U_\infty^2 s}$

$C_L$  Lift coefficient  $= \frac{L}{\frac{1}{2}\rho U_\infty^2 s}$

$C_{L_d}$  Design lift coefficient

$C_M$  Moment coefficient  $= \frac{M}{\frac{1}{2}\rho U_\infty^2 s^2}$

$C_{M_3}$  Third moment coefficient  $= \frac{M_3}{\frac{1}{2}\rho U_\infty^2 s^2}$

$D$  Drag

$L$  Lift

$M$  First moment about the leading edge

$M_3$  Third moment about the leading edge

$p$  Local static pressure

$p_\infty$  Static pressure of stream at infinity

$p_c$  Cavity pressure

$s$  Body chord length

$s'$  Dummy variable

$t$  Dummy variable

$U_\infty$  The uniform velocity at  $\infty$ , parallel to the  $x$ -axis

$u$  The  $x$ -component of the perturbation velocity

$\vec{v}$  The velocity of the fluid at any point in the flow field  $= \vec{U}_\infty + \vec{v}$

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$v$	The $y$ -component of the perturbation velocity
$\vec{v}$	The perturbation velocity
$x$	A space coordinate parallel to $U_\infty$
$x'$	Dummy variable
$x_h$	The length of the fixed portion of a flapped-plate hydrofoil
$y$	A space coordinate measured normal to the $x$ -direction
$z$	A complex variable, $z = x + iy$
$\alpha$	Operating angle of attack measured from the direction of $x$
$\alpha'$	The difference between the operating angle of attack and the design angle of attack
$\delta(x)$	The Dirac delta function
$\epsilon$	The flap angular deflection for the flapped-plate hydrofoil
$\zeta$	A complex variable, $\zeta = \xi - i\eta$
$\zeta_1$	A parameter
$\eta$	The imaginary coordinate in the complex $\zeta$ -plane
$\theta$	A parameter defined on the airfoil: $\frac{\bar{z}}{s} = \frac{1}{2}(1 - \cos \theta)$
$\theta_h$	A parameter defined for the flapped-plate hydrofoil, $\sqrt{\frac{x_h}{s}} = \frac{1}{2}(1 - \cos \theta_h)$
$v$	The complex velocity, $v = u - iv$
$\xi$	The real coordinate in the complex $\zeta$ -plane
$\rho$	The constant fluid density
$\sigma$	Cavitation number $= \frac{p_\infty - p_c}{\frac{1}{2} \rho U_\infty^2}$
$\phi$	Perturbation velocity potential such that $\vec{v} = \nabla \phi$

Subscript  $s$  Indicates quantities determined on the body surface

Subscript  $c$  Indicates quantities determined on the cavity surface

Unbarred symbols refer to cavity flow, Figure 1

Barred symbols refer to non-cavity flow, Figure 2

$O$  symbolizes "order of"



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### ABSTRACT

↙ A linearized theory is developed for steady, two-dimensional cavity flows about hydrofoil sections at zero cavitation number. The problem of calculating the flow characteristics including the hydrofoil forces and pitching moments is reduced to an equivalent problem of the classical thin airfoil theory. This equivalence is used to investigate the characteristics of hydrofoil shapes of practical importance. In particular, the lift, drag, and pitching moment due to angle of attack and flap deflection are determined. The important effect of hydrofoil shape on the cavitation drag is revealed, and a family of moderately low drag sections is specified.

The linearized theory results for the flat plate are shown to be equal to the first-order (in angle of attack) terms of the exact theory results. ↘

### INTRODUCTION

With the increasing speeds of naval vessels and underwater missiles, effects of cavitation become increasingly important. For sufficiently high speeds, the design of propellers, plane control surfaces, and main supporting foils requires information regarding the forces and moments developed on lifting surfaces with fully developed, trailing cavities. For surfaces of moderate aspect ratio with sufficiently long cavities, much of the necessary design information would be obtained from studies of two-dimensional hydrofoils operating at zero cavitation number, that is, with a pressure in the cavity equal to the pressure of the undisturbed stream. Such a study, utilizing a first-order approximation linearized theory, comprises the content of the present paper.

The actual problem being considered here may be stated in the following way: To find the lift, drag, and pitching moment about the leading edge, of two-dimensional sections immersed at an angle of attack, in a uniform, infinite, steady, inviscid stream for which it is assumed that cavitation occurs when the fluid pressure on the surface of the body becomes equal to the pressure of the undisturbed stream. For such a flow, the cavity extends to infinity. The hydrofoil sections are assumed to be of almost arbitrary shape; the only specifications are that (a) they have sharp trailing edges and sufficiently sharp noses so that cavitation occurs at both the leading and trailing edges, (b) the section is sufficiently thin so that its upper surface lies completely within the cavity which springs from the leading edge and thus does not play a role in the determination of the flow pattern, and (c) the slopes and curvatures of the sections' bottoms are sufficiently small to allow meaningful results to be obtained from the linearized theory. Although the problem is considered for all sections meeting the above specifications, the derived results are physically meaningful only for those sections for which the derived pressures on the bottom surface are nowhere less than the pressure of the stream at infinity.

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Earlier mathematical investigations which are pertinent to the present problem (see Reference 1, Chapter II) are almost all concerned with exact solutions. Of that work which is not mainly concerned with mathematical existence and uniqueness questions, the work of Levi-Civita is particularly of interest; see Chapter XII of Reference 2 for an excellent discussion. Levi-Civita considers an inverse problem and obtains results which allow the construction of flows like those considered here, but without initial specification of the hydrofoil shape. With regard to the practical solution of the present problem, there has been a little progress beyond Levi-Civita's work. As far as exact solutions in closed form are concerned, then, the general direct problem, which is nonlinear in nature, remains unsolved. This fact suggests the derivation of approximate theory. Considering the nature of the configurations involved, a first-order linearized theory identical in its approximations to the now classical thin airfoil theory, and similar to the linearized theory already developed for symmetric, two-dimensional cavity flows (see Reference 3), seems especially appropriate.

### LINEARIZED THEORY

It will be useful to consider both the linearized version of the present cavitation problem and the classical problem of thin airfoil theory, for which the solution has long been known.

Thus, consider the flows schematically illustrated in Figures 1 and 2, and let

$\vec{v}$  = velocity of fluid at any point in the flow field,  $= \vec{U}_\infty + \vec{v}$

$\vec{v}$  = perturbation velocity,

$u, v$  =  $x$  and  $y$  components, respectively, of  $\vec{v}$ ,

$U_\infty$  = uniform velocity at  $\infty$ , parallel to the  $x$  axis,

$p$  = local static pressure,

$p_\infty$  = static pressure of stream at infinity,

$p_c$  = cavity pressure,

$\sigma$  = cavitation number  $= \frac{p_\infty - p_c}{\frac{1}{2} \rho U_\infty^2}$ ,

$\rho$  = constant fluid density,

$\phi$  = perturbation velocity potential such that  $\vec{v} = \nabla \phi$ .

Subscript  $o$  indicates quantities determined on the body surface,

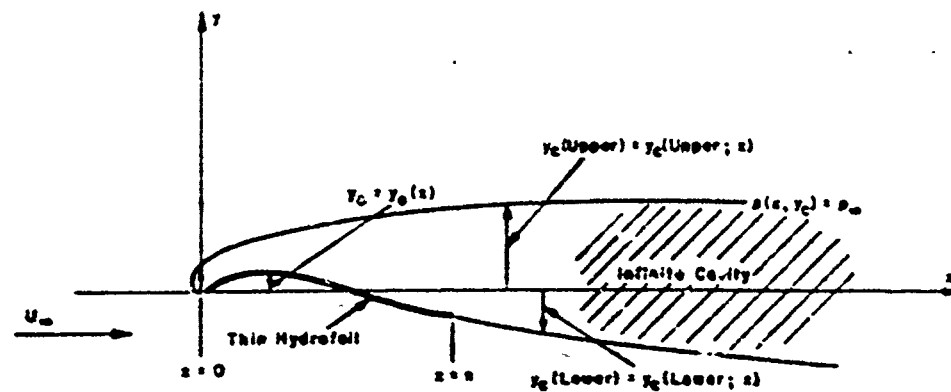
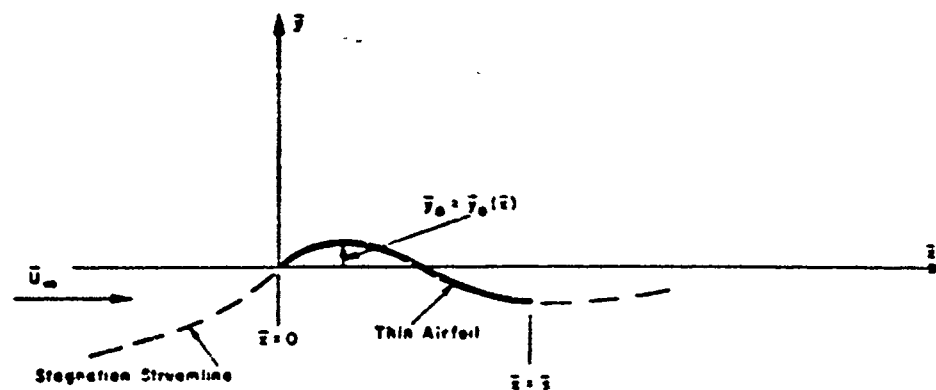
Subscript  $c$  indicates quantities determined on the cavity surface,

Unbarred symbols refer to the cavity flow, Figure 1,

Barred symbols refer to the noncavity flow, Figure 2,

$O$  symbolizes "order of."

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Figure 1 - Schematic Cavity Flow  $\sigma = 0$ Figure 2 - Schematic Noncavity Flow ( $\sigma = \infty$ )

On the bottom of the hydrofoil, the streamline slope is specified by

$$\frac{dy_0}{dx}(x) = \frac{v(x, y_0)}{U_\infty + u(x, y_0)} = \frac{v(x, y_0)}{U_\infty} \left[ 1 - \frac{u(x, y_0)}{U_\infty} + O\left(\frac{u(x, y_0)}{U_\infty}\right)^2 \right] \quad [1]$$

On the airfoil, the streamline slope is specified on the top by

$$\frac{d\bar{y}_0}{d\bar{x}}(\bar{x}) = \frac{\bar{v}(\text{upper}; \bar{x}, \bar{y}_0)}{\bar{U}_\infty} \left[ 1 - \frac{\bar{u}(\text{upper}; \bar{x}, \bar{y}_0)}{\bar{U}_\infty} + O\left(\frac{\bar{u}(\text{upper}; \bar{x}, \bar{y}_0)}{\bar{U}_\infty}\right)^2 \right]$$

and on the bottom by

$$\frac{d\bar{y}_0}{d\bar{x}}(\bar{x}) = \frac{\bar{v}(\text{lower}; \bar{x}, \bar{y}_0)}{\bar{U}_\infty} \left[ 1 - \frac{\bar{u}(\text{lower}; \bar{x}, \bar{y}_0)}{\bar{U}_\infty} + O\left(\frac{\bar{u}(\text{lower}; \bar{x}, \bar{y}_0)}{\bar{U}_\infty}\right)^2 \right] \quad [2]$$

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On the wall of the hydrofoil's cavity, the static pressure and thus the cavitation number are specified. From Bernoulli's equation it follows that

$$\sigma = \frac{2u(x, y_c)}{U_\infty} + O\left(\frac{v(x, y_c)}{U_\infty}\right)^2 = \text{constant} \quad [3]$$

It may be inferred from the Cauchy-Riemann equations that the perturbation velocity changes very slowly in space if streamline slopes and curvatures are small, so that some justification exists for satisfying linearized boundary conditions on the  $x$ - (or  $\bar{x}$ -) axis instead of on the thin bodies. At this point it is not profitable to attempt further to justify the approximations involved in the linearization, but rather to provide further justification thru a comparison that will finally be made between exact and linearized results for a particular case (pages 9 and 10).

The linearized boundary conditions become

$$\frac{dy_0}{dx}(x) = \frac{v(x, 0-)}{U_\infty} = \frac{1}{U_\infty} \frac{\partial \phi}{\partial y}(x, 0-); \quad 0 < x < s \quad [1a]$$

$$\sigma = 0 = \frac{2u(x, 0-)}{U_\infty} = \frac{2}{U_\infty} \frac{\partial \phi}{\partial x}(x, 0-); \quad x > s \quad [3a]$$

$$\sigma = 0 = \frac{2u(x, 0+)}{U_\infty} = \frac{2}{U_\infty} \frac{\partial \phi}{\partial x}(x, 0+); \quad x > 0$$

$$\frac{d\bar{y}_0}{d\bar{x}}(\bar{x}) = \frac{\bar{v}(\bar{x}, 0)}{\bar{U}_\infty} = \frac{1}{\bar{U}_\infty} \frac{\partial \bar{\phi}}{\partial \bar{y}}(\bar{x}, 0); \quad 0 < \bar{x} < \bar{s} \quad [2a]$$

Finally, then, the linearized problems may be stated: To find the harmonic functions  $\phi(x, y)$  and  $\bar{\phi}(\bar{x}, \bar{y})$  whose gradients in the limit vanish everywhere on a circle of sufficiently large radius about the origin, which satisfy the boundary conditions of Equations [1a], [2a], [3a] (which are incorporated in Figures 3 and 4) and which, for the sake of physical reality, produce smooth flow at the points  $(s, 0-)$  and  $(\bar{s}, 0)$ . In Figure 4, the asymmetry of  $\bar{\phi}(\bar{x}, \bar{y})$ , which follows from the definition of  $\bar{\phi}(\bar{x}, \bar{y})$  just given, is used to specify boundary conditions on the entire  $\bar{x}$ -axis.

The problem for  $\bar{\phi}(\bar{x}, \bar{y})$  is essentially solved through the observation that its boundary conditions may be satisfied by a suitable distribution of vortex singularities placed on the interval  $0 < \bar{x} < \bar{s}$ . The strength of the distributed vortex singularities is proportional to  $\bar{u}(\bar{x}, 0+)$ . An integral equation is then obtained which relates  $\bar{u}(\bar{x}, 0+)$  to the airfoil shape. It is

$$\bar{v}(\bar{x}, 0) = \bar{U}_\infty \frac{d\bar{y}_0}{d\bar{x}}(\bar{x}) = \frac{1}{\pi} \int_0^{\bar{s}} \frac{\bar{u}(\bar{x}', 0+)}{\bar{x}' - \bar{x}} d\bar{x}' \quad [4]$$

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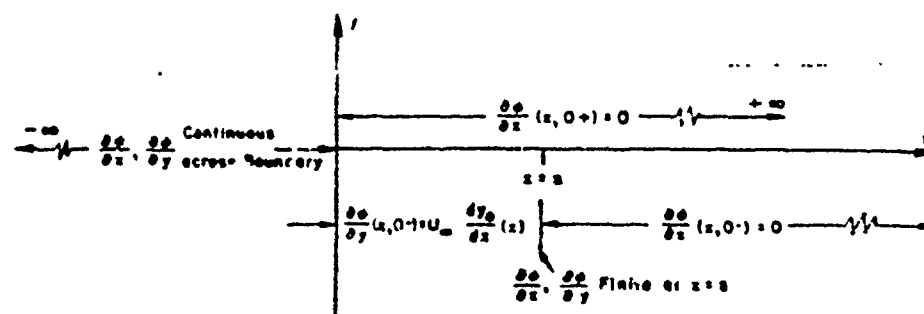
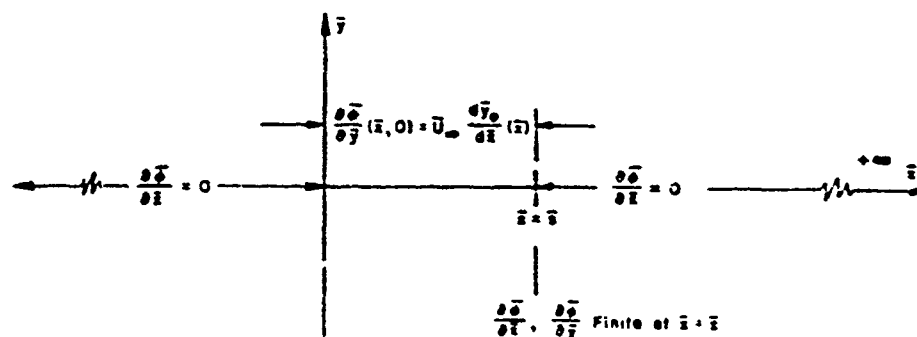
Figure 3 - The Linearized Boundary Conditions for a Cavitating Hydrofoil,  $\sigma = 0$ 

Figure 4 - The Linearized Boundary Conditions for an Airfoil. No Cavitation.

The integral Equation [4] is conveniently solved by Glauert's method (see Reference 4, Chapter VII). Useful results of the solution are collected for reference in Appendix A.

#### THE HYDROFOIL-AIRFOIL EQUIVALENCE

It is now to be shown that for every cavitating hydrofoil problem of the type just described, an intimately related thin airfoil problem exists whose solution may readily be converted into the solution of the hydrofoil problem.

First, recall that the complex velocity is an analytic function of the complex space coordinate:

$$u(x, y) - iv(x, y) = f(z + iy) = f(z)$$

and that the function  $g(\zeta)$  created from  $f(z)$  by a conformal transformation of the  $z$ -space to a new, say,  $\zeta$ -space is also a complex velocity.

Then consider the transformation

$$\sqrt{z} = -\zeta = -(\xi + i\eta) \quad [5]$$

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which transforms the top of the positive  $x$ -axis into the negative  $\zeta$ -axis, the bottom of the positive  $x$ -axis into the positive  $\zeta$ -axis, and the entire  $x$ -plane, exclusive of the cut positive  $x$ -axis, into the lower half of the  $\zeta$ -plane.

The flow problem, schematically illustrated in Figure 3, is shown as a complex variable problem in the original  $x$ -plane in Figure 5, and is shown in the transformed  $\zeta$ -plane in Figure 6.

It follows from a comparison of Figures 4 and 6, that the problem represented in Figure 3 is exactly a thin airfoil problem. The airfoil equivalent to any given hydrofoil is one such that (using the initial notation for the airfoil flow):

$$\bar{v}(\bar{z}) = v(\bar{z}^2); \quad v(x) = \bar{v}(\sqrt{x})$$

or,

[6]

$$\frac{d\bar{y}_0}{d\bar{z}}(\bar{z}) = \frac{dy_0}{dz}(\bar{z}^2); \quad \frac{dy_0}{dz}(x) = \frac{d\bar{y}_0}{d\bar{z}}(\sqrt{x})$$

$$\bar{u}(\bar{z}) = u(\bar{z}^2); \quad u(x) = \bar{u}(\sqrt{x})$$

[7]

$$\text{and } \bar{U}_\infty = U_\infty$$

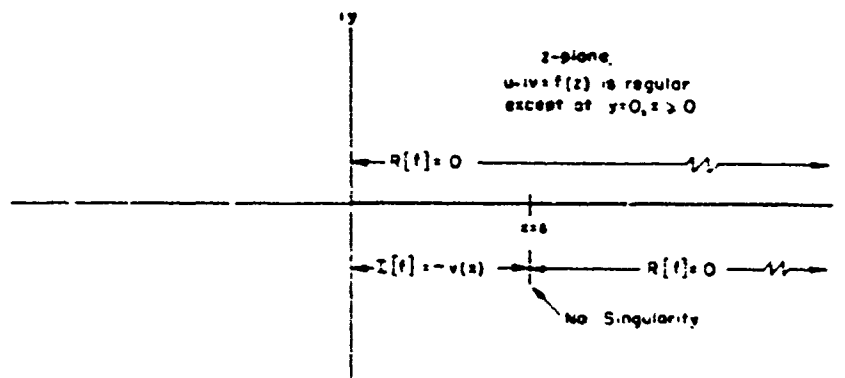


Figure 5 - The Hydrofoil Problem in  $x$ -space;

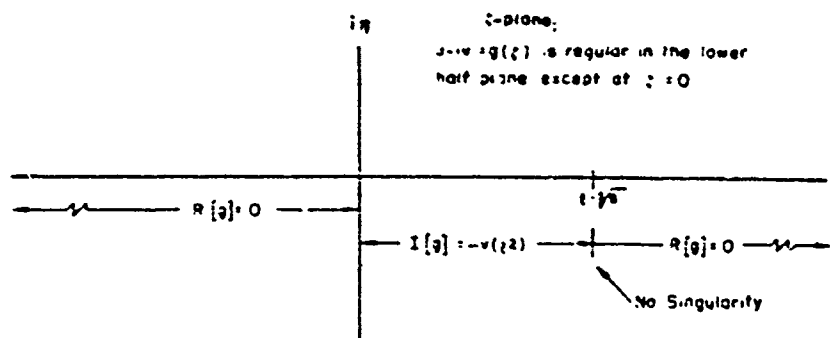


Figure 6 - The Hydrofoil Problem in  $\zeta$  Space,  $\sqrt{x} = -\zeta$

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As examples:

- a. The flat plate hydrofoil of chord  $s$  at an angle of attack  $\alpha$ ,  $[(dy_0/dx)(x) = -\alpha]$ , is equivalent to the flat plate airfoil of chord  $\sqrt{s}$  at an angle of attack  $\alpha$ ,  $[(dy_0/d\bar{x})(\bar{x}) = -\alpha]$ .
- b. The hydrofoil of shape  $y_0 = ax + bx^2$ ,  $(0 < x < 1)$ ,  $[(dy_0/dx)(x) = a + 2bx]$  is equivalent to the airfoil of shape  $\bar{y}_0 = a\bar{x} + 2b/3\bar{x}^3$ ,  $(0 < \bar{x} < 1)$ ,  $[(d\bar{y}_0/d\bar{x})(\bar{x}) = a + 2b\bar{x}^2]$ .

As a consequence of the hydrofoil-airfoil equivalence, the lift, drag, and pitching moment of a hydrofoil correspond to certain characteristics of the equivalent airfoil.

For the hydrofoil:

$$\text{The lift } L = \int_0^s [p(x, 0-) - p(x, 0+)] dx \quad [8]$$

$$\text{The drag } D = - \int_0^s \frac{dy_0}{dx}(x) [p(x, 0-) - p(x, 0+)] dx \quad [9]$$

$$\begin{aligned} \text{The moment } M \text{ about the leading edge (+ counterclockwise)} \\ = \int_0^s x [p(x, 0-) - p(x, 0+)] dx \end{aligned} \quad [10]$$

Using the linearized Bernoulli equation and nondimensionalizing, these expressions become

$$C_L = \frac{L}{\frac{1}{2} \rho U_\infty^2 s} = - \int_0^s \frac{2u(x, 0-)}{s U_\infty} dx \quad [8a]$$

$$\begin{aligned} C_D = \frac{D}{\frac{1}{2} \rho U_\infty^2 s} &= \int_0^s 2 \frac{dy_0}{dx}(x) \cdot \frac{u(x, 0-)}{s U_\infty} dx \\ &= \int_0^s 2 \frac{v(x, 0-)}{U_\infty} \cdot \frac{u(x, 0-)}{U_\infty} \frac{dx}{s} \end{aligned} \quad [9a]$$

$$C_M = \frac{M}{\frac{1}{2} \rho U_\infty^2 s^2} = - \int_0^s \frac{2x}{s^2} \frac{u(x, 0-)}{U_\infty} dx \quad [10a]$$

These coefficients may be expressed in terms of quantities in the airfoil plane:

$$C_L = - \int_0^{\sqrt{s}} \frac{4\bar{x} \bar{u}(\bar{x}, 0-)}{s U_\infty} d\bar{x} \quad [30]$$

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$$C_D = \int_0^{\sqrt{s}} \frac{\bar{u}(\bar{x}, 0-) \bar{u}(\bar{x}, 0+)}{s U_\infty^2} d\bar{x} \quad [9b]$$

$$C_M = - \int_0^{\sqrt{s}} \frac{4\bar{x}^3}{s^2} \frac{\bar{u}(\bar{x}, 0-)}{U_\infty} d\bar{x} \quad [10b]$$

The expression for the drag coefficient may be further reduced by using the result contained in Equation (4):

$$C_D = - \int_0^{\sqrt{s}} \frac{4\bar{x} \bar{u}(\bar{x}, 0-)}{\pi s U_\infty^2} d\bar{x} \int_0^{\sqrt{s}} \frac{\bar{u}(\bar{x}', 0-)}{\bar{x}' - \bar{x}} d\bar{x}' = - \int_0^{\sqrt{s}} \frac{4\bar{x} \bar{u}(\bar{x}, 0-)}{\pi s U_\infty^2} d\bar{x} \int_0^{\sqrt{s}} \frac{\bar{u}(\bar{x}, 0-)}{\bar{x} - \bar{x}'} d\bar{x}' \quad [9c]$$

Using the identity  $\frac{\bar{x}}{\bar{x} - \bar{x}'} = \frac{\bar{x}'}{\bar{x} - \bar{x}'} + 1$

$$C_D = + \int_0^{\sqrt{s}} \frac{4\bar{x} \bar{u}(\bar{x}, 0-)}{\pi s U_\infty^2} d\bar{x} \int_0^{\sqrt{s}} \frac{\bar{u}(\bar{x}, 0-)}{\bar{x} - \bar{x}'} d\bar{x}' + \int_0^{\sqrt{s}} \frac{4\bar{x} \bar{u}(\bar{x}, 0-)}{\pi s U_\infty^2} d\bar{x} \int_0^{\sqrt{s}} \bar{u}(\bar{x}, 0-) d\bar{x}' \quad [9d]$$

or

$$2\pi C_D = \left[ \frac{2}{\sqrt{s} U_\infty} \int_0^{\sqrt{s}} \bar{u}(\bar{x}, 0-) d\bar{x} \right]^2 \quad [9e]$$

For the equivalent airfoil:

$$\text{The lift } L = \int_0^{\sqrt{s}} [\bar{p}(\bar{x}, 0-) - \bar{p}(\bar{x}, 0+)] d\bar{x} \quad [11]$$

The moment  $\bar{M}$  about the leading edge (- counterclockwise)

$$= \int_0^{\sqrt{s}} \bar{x} [\bar{p}(\bar{x}, 0-) - \bar{p}(\bar{x}, 0+)] d\bar{x} \quad [12]$$

$$\text{The third moment } \bar{M}_3 = \int_0^{\sqrt{s}} \bar{x}^3 [\bar{p}(\bar{x}, 0-) - \bar{p}(\bar{x}, 0+)] d\bar{x} \quad [13]$$

Again using the linearized Bernoulli equation and nondimensionalizing, these expressions become:

$$\bar{C}_L = \frac{\bar{L}}{\frac{1}{2} \rho U_\infty^2 \sqrt{s}} = - \int_0^{\sqrt{s}} \frac{4 \bar{u}(\bar{x}, 0-)}{\sqrt{s} U_\infty} d\bar{x} \quad [11a]$$

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$$\bar{C}_M = \frac{\bar{M}_1}{\frac{1}{2} \rho U_\infty^2 s} = - \int_0^{\sqrt{s}} \frac{4\bar{x} \bar{u}(\bar{x}, 0-)}{s L_\infty} d\bar{x} \quad [12a]$$

$$\bar{C}_{M_3} = \frac{\bar{M}_3}{\frac{1}{8} \rho U_\infty^2 s^2} = - \int_0^{\sqrt{s}} \frac{4\bar{x}^3 \bar{u}(\bar{x}, 0-)}{s^2 U_\infty} d\bar{x} \quad [13a]$$

A comparison of Equations [8b], [9e], and [10b] with Equations [11a], [12a] and [13a] yields the following identities:

$$C_L = \bar{C}_M \quad [14]$$

$$C_M = \bar{C}_{M_3} \quad [15]$$

$$C_D = \frac{1}{8\pi} \bar{C}_L^2 \quad [16]$$

These results, together with Equation [6] which defines the airfoil equivalent to a given hydrofoil, completely accomplish the reduction of the hydrofoil problem to an airfoil problem. They will be used in the following sections to study the characteristics of the supercavitating hydrofoil.

#### THE FLAT-PLATE HYDROFOIL

The case of the cavitating flat plate at an angle of attack  $\alpha$  is particularly interesting since the exact solution has long been known (see Reference 5, Chapter IV) and because it provides information on forces and moments due solely to angle of attack.

The exact results for lift, drag, and pitching moment are:

$$C_L(\text{exact}) = \frac{2\pi \sin \alpha \cos \alpha}{4 + \pi \sin \alpha} = \frac{\pi \alpha}{2} + O(\alpha^2) \quad [17]$$

$$C_M(\text{exact}) = \left( \frac{2\pi \sin \alpha}{4 + \pi \sin \alpha} \right) \left( \frac{1}{2} - \frac{3 \cos \alpha}{4[4 + \pi \sin \alpha]} \right) = \frac{5\pi \alpha}{32} + O(\alpha^2) \quad [18]$$

$$C_D(\text{exact}) = C_L(\text{exact}) \cdot \tan \alpha = \frac{\pi \alpha^2}{2} + O(\alpha^3) \quad [19]$$

According to Equation [6], and as has been noted, the flat-plate hydrofoil at angle of attack  $\alpha$  is equivalent to a flat-plate airfoil at angle of attack  $\alpha$ . It is easily shown, using results of the thin airfoil theory, (see Appendix A) that

$$C_L(\text{linearized}) = \bar{C}_M(\text{linearized}) = \frac{\pi \alpha}{2} \quad [17a]$$

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$$C_M(\text{linearized}) = \bar{C}_{M_3}(\text{linearized}) = \frac{5\pi\alpha}{32} \quad [13a]$$

$$C_D(\text{linearized}) = \frac{1}{8\pi} \bar{C}_L^2(\text{linearized}) = \frac{(2\pi\alpha)^2}{8\pi} = \frac{\pi\alpha^2}{2} \quad [19a]$$

The shape of the cavity may also be determined. It is shown in Appendix B that a foil of unit chord:

$$\begin{aligned} y_c(\text{linearized; upper}) &= \alpha \left[ -x + \frac{1}{2}(1 + 2\sqrt{x}) \sqrt{x + \sqrt{x}} \right. \\ &\quad \left. + \frac{1}{4} \ln(1 + 2\sqrt{x} - 2\sqrt{x + \sqrt{x}}) \right], \quad x > 0 \\ y_c(\text{linearized; lower}) &= \alpha \left[ -x + \frac{1}{2}(2\sqrt{x} - 1) \sqrt{x - \sqrt{x}} \right. \\ &\quad \left. - \frac{1}{4} \ln(-1 + 2\sqrt{x} + 2\sqrt{x - \sqrt{x}}) \right], \quad x > 1 \end{aligned} \quad [20]$$

and,

$$y_c(\text{exact}) = y_c(\text{linearized}) + O(\alpha^2) \quad [20a]$$

The fact that the linearized theory, as applied here to the case of the flat plate gives results for forces, moments, and cavity shapes that are identical with the leading term in the exact solution expressions for corresponding quantities, is an important justification for the linearized theory and, in addition, provides a check on the correctness of the hydrofoil-airfoil equivalence results.

#### THE FLAPPED-PLATE HYDROFOIL

Because of its usefulness as a control surface, the configuration consisting of a hydrofoil with a hinged or flapped after portion is of particular interest. As a particular case, and one revealing the effect of the flap size on the flap effectiveness, the flapped-plate hydrofoil, as shown schematically in Figure 7, will be discussed.

It follows from Equations [17] and [18] that

$$A_0 = \epsilon \left( \frac{\pi - \theta_A}{\pi} \right) \quad [21]$$

$$A_n = \frac{2\epsilon}{\pi n} \sin n\theta_A \quad [22]$$

where

$$\frac{z_A}{s} = \frac{1}{2}(1 - \cos \theta_A) - \sqrt{\frac{z_A}{s}} \quad [23]$$

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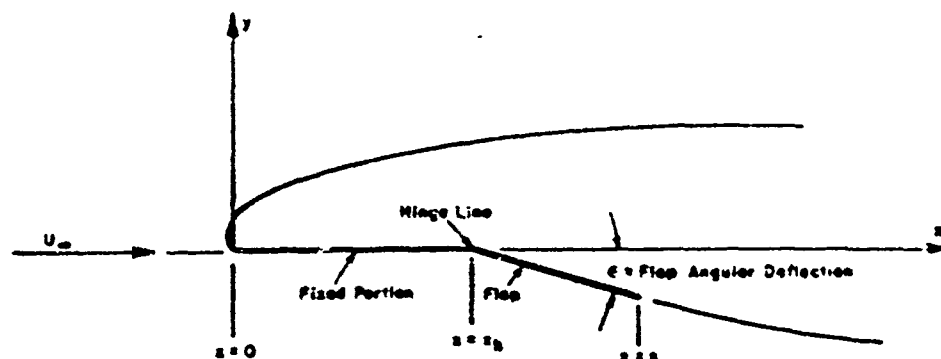


Figure 7 - Flapped-Plate Hydrofoil

Then, using Equations [A3], [A4], [A5] and Equations [14], [15], [16],

$$C_L = \bar{C}_M = \frac{\pi}{2} \left( A_0 + A_1 - \frac{A_2}{2} \right) - \frac{e}{2} [x - \theta_h + \sin \theta_h (2 - \cos \theta_h)] \quad [24]$$

$$C_M = \bar{C}_{M_3} = \frac{\pi}{32} \left( 5A_0 + 7A_1 - 7A_2 + 3A_3 - \frac{A_4}{2} \right) \quad [25]$$

$$= \frac{e}{32} [5(x - \theta_h) + 14 \sin \theta_h - 7 \sin 2\theta_h + 2 \sin 3\theta_h - \frac{1}{4} \sin 4\theta_h]$$

$$C_D = \frac{1}{8\pi} \bar{C}_L^2 = \frac{\pi}{2} \left( A_0 + \frac{A_1}{2} \right)^2 = \frac{e^2}{2\pi} [x - \theta_h + \sin \theta_h]^2 \quad [26]$$

The quantities  $(dC_L)/(de)$  and  $(dC_M)/(de)$  are shown as a function of flap chord ratio in Figure 8. It is important to note that the lift effectiveness of all flaps larger than 41 percent chord is as great as or greater than the lift effectiveness of an unflapped foil. A 75 percent flap, for which the lift effectiveness is an optimum, has a lift effectiveness 114 percent of the unflapped foil effectiveness. This is in contrast with the situation for noncavitating airfoils, for which the flap effectiveness continually increases with increasing flap chord ratio.

The lift/drag ratio of a flapped foil for a given lift coefficient is greater the smaller the flap-chord ratio. In Figure 9 this ratio is shown as a function of lift coefficient for an unflapped plate and for a 25 percent flapped plate for which  $L/D = 5.21, C_L$ . The flapped plate is seen to be decidedly superior. It is an important result, demonstrated by this example, that the cavitation drag of a hydrofoil operating at a given lift coefficient is very much dependent on the hydrofoil shape.

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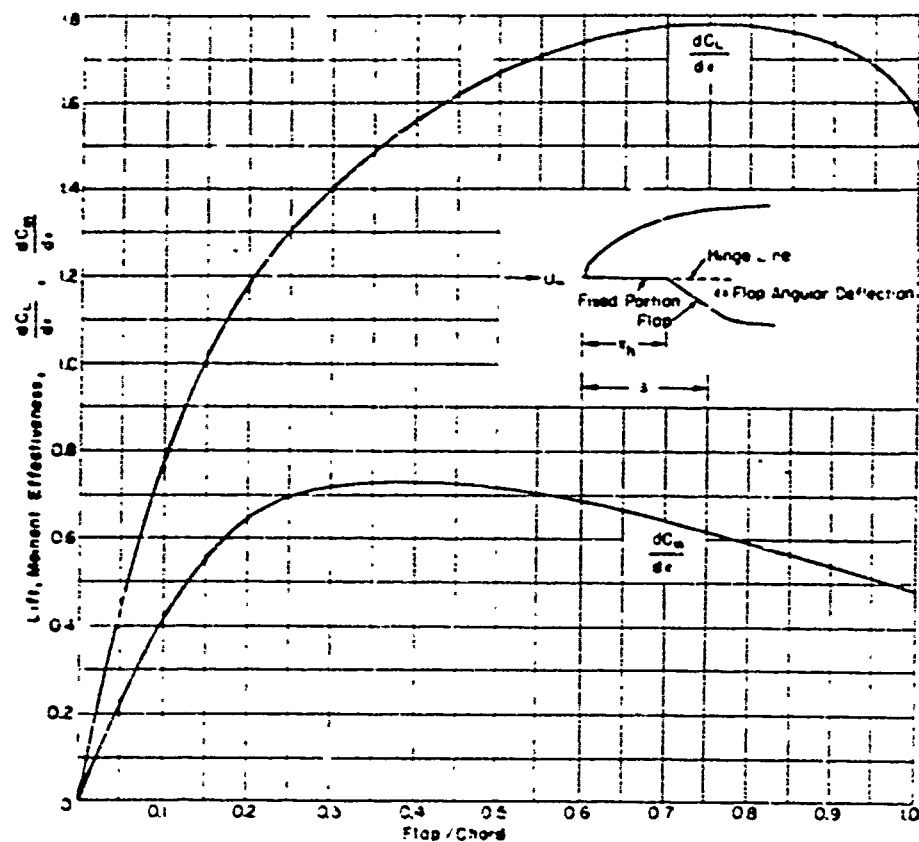


Figure 5 - Lift and Moment Effectiveness for Flapped-Plate Hydrofoil

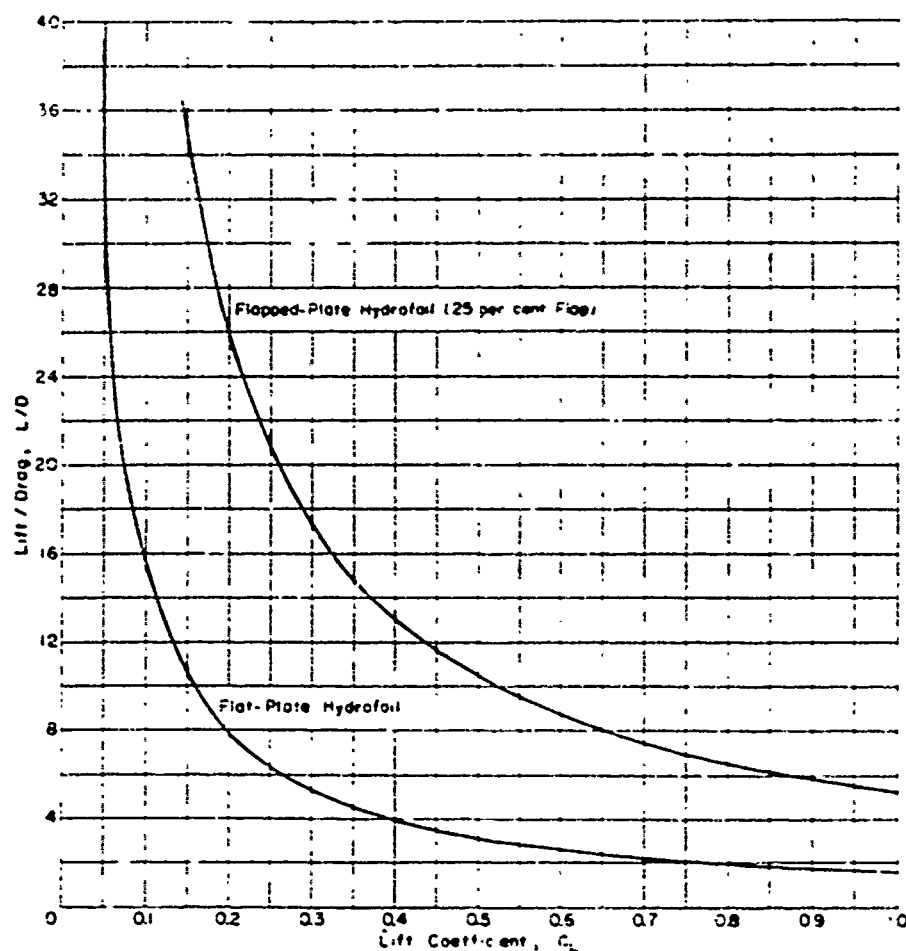
#### LOW-DRAG HYDROFOILS

The specification of hydrofoils with low cavitation drag is a problem of great practical importance. Because the drag may be several times larger than necessary on an improperly designed configuration, the success of high-speed hydrofoil boats and, particularly, super-cavitating screws may well depend upon the proper choice of the hydrofoil sections.

It is of interest first to determine the minimum  $C_D$  (the lift coefficient being specified) for the class of all possible cavitating hydrofoils. In view of the hydrofoil-airfoil equivalence, the problem becomes one of determining the minimum  $\bar{C}_L$  (the moment coefficient being specified) for the class of all possible airfoils on whose lower surface the pressure is not here

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Figure 3 -  $L/D$  vs.  $C_L$  for Flat-Plate and Flapped-Plate Hydrofoils

less than the free stream pressure. Low drag equivalent airfoils will thus have centers of pressure relatively far aft of the leading edge. The optimum airfoil pressure distribution can be immediately specified. It is shown in Figure 10 where  $\delta(\bar{x})$  is the Dirac delta function defined such that

$$\int_{-\infty}^{\infty} \delta(\bar{x} - \bar{x}_1) d\bar{x} = 1 \text{ and } \delta(\bar{x} - \bar{x}_1) = 0 \text{ for } \bar{x} \neq \bar{x}_1$$

The question of the practical possibility of obtaining such a pressure distribution need not be considered. The important thing is that this case does furnish a lower bound for the hydrofoil drag which may be used as a measure of drag performance.

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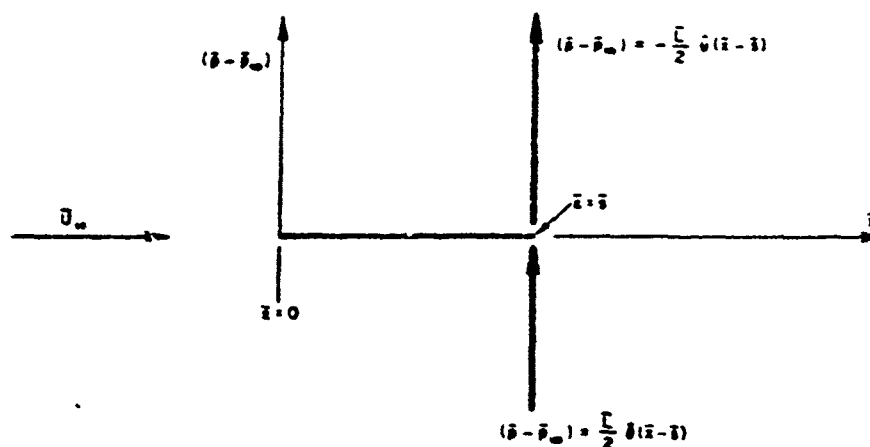


Figure 10 - Optimum Low-Drag Pressure Distribution on Equivalent Airfoil

For this optimum pressure distribution,

$$\bar{C}_M = \frac{\bar{M}}{\frac{1}{2} \rho \bar{U}_\infty^2 \bar{s}^2} = \frac{\bar{s} \bar{L}}{\frac{1}{2} \rho \bar{U}_\infty^2 \bar{s}^2} = \bar{C}_L \quad [27]$$

Then,

$$C_L = \bar{C}_L \quad [28]$$

$$C_D = \frac{1}{8\pi} \bar{C}_L^2 \quad [29]$$

$$\left( \frac{C_L}{C_D} \right)_{\text{opt.}} = \frac{8\pi}{C_L} \quad [30]$$

This optimum may be compared to the lift drag ratio of the flat plate, which, as has already been indicated, is an inferior section.

$$\left( \frac{C_L}{C_D} \right)_{\text{flat plate}} = \frac{\pi}{2C_L} \quad [31]$$

It is of importance now to specify a family of hydrofoil sections with relatively low-drag characteristics. Such a family is one whose equivalent airfoil family is described by

$$\frac{d\bar{\phi}}{ds} = A_1 \left( \cos \theta - \frac{1}{2} \cos 2\theta \right) = \frac{A_1}{2} \left[ 1 + \frac{4\pi}{s} - 8 \left( \frac{\pi}{s} \right)^2 \right] \quad [32]$$

These shapes were chosen from a more general family in such a way that they have a minimum drag and still have pressure distributions such that the bottom pressures are at no

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is not less than the free stream pressure. The general family dealt with is one for which the equivalent airfoil family consists of shapes such that  $dy_0/dx = -A_0 + A_1 \cos \theta + A_2 \cos 2\theta$  (see Equation [46], Appendix A).

For these low-drag shapes,

$$\bar{C}_L = \pi A_1 \quad [33]$$

$$\bar{C}_M = \frac{5\pi A_1}{8} \quad [34]$$

$$\bar{C}_{M_3} = \frac{21\pi A_1}{64} \quad [35]$$

The shape of the corresponding hydrofoil family is

$$\frac{dy_0}{dx} = \frac{A_1}{2} \left( 1 + 4\sqrt{\frac{x}{s}} - 8\frac{x}{s} \right) \quad [36]$$

or

$$\frac{y_0}{s} = \frac{A_1}{2} \left[ \frac{x}{s} + \frac{8}{3} \left( \frac{x}{s} \right)^{3/2} - 4 \left( \frac{x}{s} \right)^2 \right] \quad [37]$$

for which

$$C_L = \bar{C}_L = \frac{5\pi A_1}{8} \quad [38]$$

$$C_M = \bar{C}_{M_3} = \frac{21\pi A_1}{64} \quad [39]$$

$$C_D = \frac{1}{8\pi} (\bar{C}_L)^2 = \frac{\pi A_1^2}{8} \quad [40]$$

$$\frac{C_L}{C_D} = \frac{5\pi}{8\pi} \quad (\text{when operating at design } C_L) \quad [41]$$

Equation [37] may be rewritten

$$\frac{y_0}{s} = \frac{4C_{L_d}}{5\pi} \left[ \left( \frac{x}{s} \right) + \frac{8}{3} \left( \frac{x}{s} \right)^{3/2} - 4 \left( \frac{x}{s} \right)^2 \right] \quad [37a]$$

where  $C_{L_d}$  is the design lift coefficient. The shape of these sections is shown in Figure 11 where  $(y_0/s)/(4C_{L_d}/5\pi)$  is plotted as a function of  $(x/s)$ .

Comparison of the drag characteristics of these low-drag airfoils with those of a flat plate reveal that for a given lift coefficient they have only one sixth the drag of a flat plate. Comparison with the optimum case (Equation [30]) indicates that even this low-drag family may be improved upon. Such a development is not to be attempted here.

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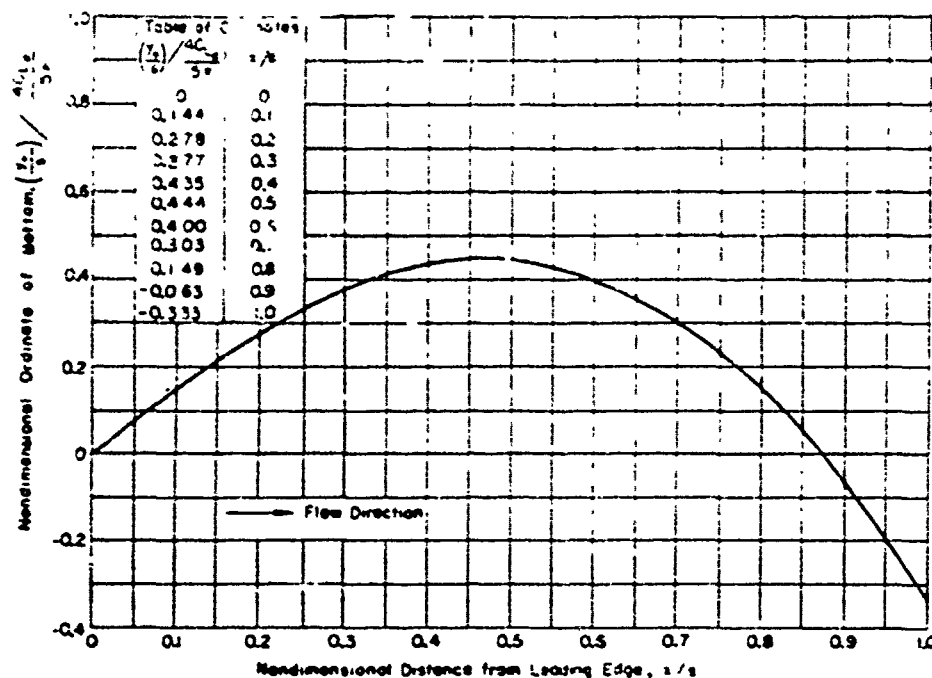


Figure 11 - Low-Drag Hydrofoil Sections

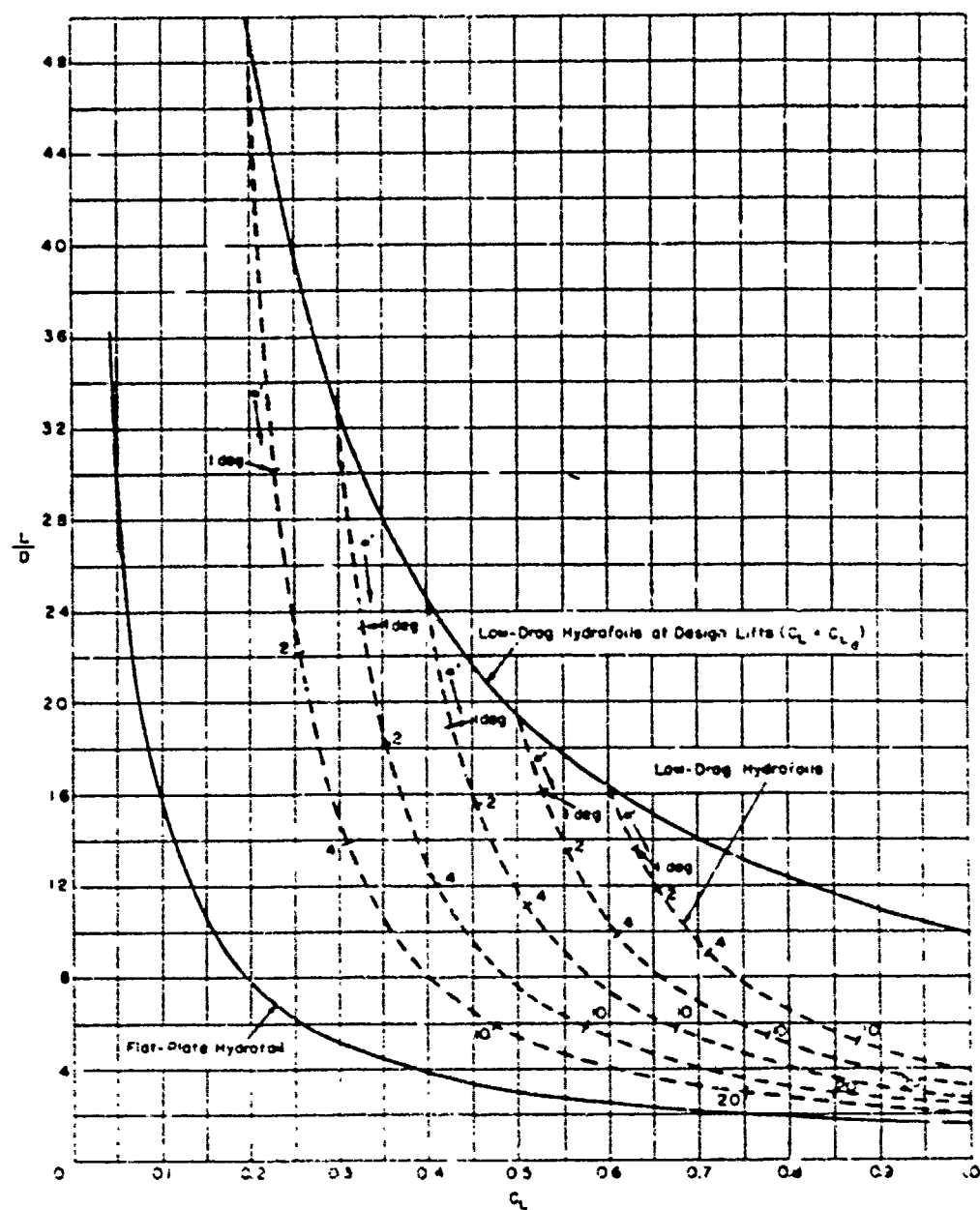
It is to be noted that the lift/drag ratio calculated (Equation [41]) is only obtained when the foil is operating at its design lift coefficient, for which, incidentally, the flow enters the leading edge without stagnating (so-called "shock-free" entry). For angles of attack less than the design angle, the foil will probably cavitate on the lower side and the lift will probably be reversed. It may thus be desirable to operate a foil at some angle of attack slightly larger than the design angle. For such a case it may be shown that

$$\frac{C_L}{C_D} = \frac{\pi}{2} \frac{C_L}{\left(C_L - \frac{3}{5} C_{L_d}\right)^2} = \frac{\alpha' + \frac{2}{5} C_{L_d}}{\left(\alpha' + \frac{4}{5} C_{L_d}\right)^2} \quad [42]$$

where  $\alpha'$  is the difference between the operating angle of attack and the design angle of attack. These ratios are shown for several members of the low-drag family in Figure 12. Large increases in drag result when foils are operated at angles of attack substantially larger than the design angles. Nevertheless, cavitation drags equal to or even less than the magnitude of the viscous drag of one wetted surface can probably be obtained in a practical design. Thus, from the drag point of view, supercavitating configurations may be designed with characteristics not inferior to those of noncavitating configurations.

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Figure 12 -  $L/D$  vs  $C_L$  for Low-Drag Hydrofoil Family

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### SUMMARY

1. The linearized theory is a meaningful first-order theory for calculating the characteristics of supercavitating hydrofoils at small angles of attack. The justification for this conclusion lies in the result that in the case of a flat plate at zero cavitation number, the linearized result for cavity shape, foil forces, and pitching moment is actually the first-order term in an expansion of the exact solution in powers of the angle of attack.

2. The problem of determining the zero cavitation number characteristics of an arbitrary hydrofoil section can be reduced in a simple manner to an equivalent thin airfoil problem. Simple relationships exist between the lift, drag, and pitching moment of the hydrofoil, and the moment, lift, and third moment of the equivalent airfoil. Effective and practical existing methods may be used to solve the thin airfoil problem.

3. The lift curve and moment curve slopes for a flat plate at zero cavitation number are, respectively,  $\pi/2$  and  $5\pi/32$ , as compared with the corresponding values for the noncavitating flat plate of  $2\pi$  and  $\pi/2$ . These values also represent the lift and moment effectiveness with respect to angle of attack of cambered sections.

4. At zero cavitation number, the lift effectiveness (with respect to flap deflection) of all flaps larger than 41 percent chord is as great or greater than the lift effectiveness of an unflapped foil (flat plate).

5. The cavitation drag accompanying a given lift is very much dependent on the hydrofoil shape. An absolute minimum drag exists for any given lift. All practical hydrofoils must have drags larger than this absolute minimum, which is  $(C_D)_{\min} = (C_L)^2/\pi\alpha$ .

6. For all size flaps, the generation of lift by flap deflection results in less drag than the generation of lift by angle of attack changes. Thus, from the cavitation drag standpoint, the flat plate is an inferior hydrofoil section.

7. The generation of lift by the proper use of camber results in less drag than the generation of lift by flap deflection for practical size flaps.

8. Cambered hydrofoils may be designed for drag operation, for which the cavitation drag at moderate lift coefficients is approximately the same as the viscous drag of one wetted surface. Thus, supercavitating configurations may probably be designed with drag characteristics not essentially inferior to those of noncavitating configurations.

9. A family of relatively low-drag hydrofoils and their theoretical characteristics are presented in the present paper.

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## APPENDIX A

## USEFUL RESULTS OF THE LINEARIZED AIRFOIL THEORY

The notation of Figure 2 will be used: in particular, the airfoil ordinate  $\bar{y}_0(\bar{x})$  is measured normal to the free stream direction. This notation differs from that of Glauert (Reference 4, Chapter VII) where the airfoil ordinate is measured normal to the chord line and accounts for those differences between the presentation below and that of Glauert.

If

$$\bar{x} = \frac{\bar{c}}{2}(1 - \cos \theta) \quad (1)$$

and it is assumed that

$$\bar{u}(\bar{x}, 0) = \bar{U}_\infty \left\{ A_0 \cos \frac{\theta}{2} + \sum_{n=1}^{\infty} A_n \sin n\theta \right\} \quad (2)$$

then it follows that

$$\bar{C}_L = \frac{\bar{L}}{\frac{1}{2} \rho \bar{U}_\infty^2 \bar{c}} = 2\pi \left( A_0 + \frac{A_1}{2} \right) \quad (A3)$$

$$\bar{C}_M = \frac{\bar{M}}{\frac{1}{4} \rho \bar{U}_\infty^2 \bar{c}^2} = \frac{\pi}{2} \left( A_0 + A_1 - \frac{A_2}{2} \right) \quad (A4)$$

$$\bar{C}_{M_3} = \frac{\bar{M}_3}{\frac{1}{8} \rho \bar{U}_\infty^2 \bar{c}^4} = \frac{\pi}{32} \left( 5A_0 + 7A_1 - 7A_2 + 3A_3 - \frac{A_4}{2} \right) \quad (A5)$$

It also follows from Equation (4) that

$$\frac{d\bar{y}_0}{d\bar{x}}(\theta) = -A_0 + \sum_{n=1}^{\infty} A_n \cos n\theta \quad (A6)$$

where

$$A_0 = -\frac{1}{\pi} \int_0^\pi \frac{d\bar{y}_0}{d\bar{x}}(\theta) d\theta \quad (A7)$$

$$A_n = \frac{2}{\pi} \int_0^\pi \frac{d\bar{y}_0}{d\bar{x}}(\theta) \cos n\theta d\theta \quad (A8)$$

The thin airfoil shape being given, the above relations permit the calculation of pressure distribution and resultant forces and moments.

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# APPENDIX B

## WAKE SHAPE ACCORDING TO LINEARIZED AND EXACT THEORIES

### LINEARIZED THEORY

According to the assumptions of the linearized theory and using the notation of Figure 1,

$$\frac{dy_e(\text{upper}; x)}{dx} = \frac{v'(x, 0+)}{U_\infty} \quad [B1]$$

$$\frac{dy_e(\text{lower}; x)}{dx} = \frac{v(x, 0-)}{U_\infty}$$

from which it follows

$$y_e(\text{upper}; x) = \frac{1}{U_\infty} \int_0^x v(t, 0+) dt \quad [B2]$$

$$y_e(\text{lower}; x) = \frac{1}{U_\infty} \int_0^x v(t, 0-) dt$$

The problem of finding  $v(t, 0+)$  and  $v(t, 0-)$  for the hydrofoil may be transformed into the problem of finding  $\bar{v}(\bar{t}, 0)$  in the airfoil plane. Making use of the transformation, Equation [5], Equation [B2] then becomes

$$y_e(\text{upper}; x) = \frac{2}{U_\infty} \int_0^{\bar{x}=\sqrt{x}} \bar{v}(\bar{t}, 0) \bar{t} d\bar{t} \quad [B3]$$

$$y_e(\text{lower}; x) = \frac{2}{U_\infty} \int_0^{\bar{x}=\sqrt{x}} \bar{v}(\bar{t}, 0) \bar{t} d\bar{t}$$

The problem of finding  $\bar{v}(\bar{t}, 0)$  for a flat-plate airfoil of unit chord  $\alpha$ ; angle of attack  $\alpha$  may be solved by a mapping technique. The image of the  $\bar{x}$ -axis in the conjugate complex velocity plane is a polygon (see Figure B1). It may be mapped onto its image in the physical plane by making use of a Schwarz-Christoffel transformation. The mapping function is then the solution of the present problem since it provides a relation between the velocity components and the physical space coordinates.

As the polygon in the complex velocity plane is traversed from  $B$  to  $C$  to  $D$  to  $A$  to  $B$ , the region on the right is to be transformed into the lower half of the physical plane. The vertices of the polygon  $BCDAB$  are to be mapped into the points 0 and 1 in the physical plane. The interior angle at  $C$  is  $\pi/2$ ; the interior angle at  $B$  is  $-\pi/2$ . The mapping function is then:

$$\frac{dz}{d\zeta} = \frac{1}{\zeta^{3/2} \sqrt{\zeta^2 - 1}} \quad [B4]$$

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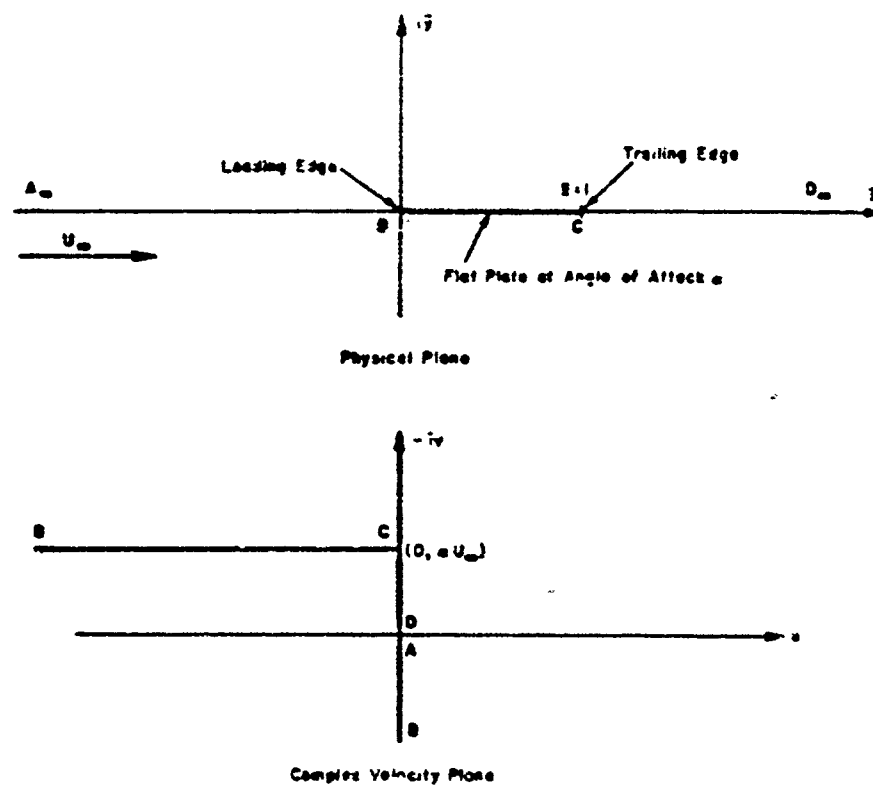


Figure B1 - Flat Plate Airfoil in Physical and Complex Velocity Planes

or

$$v = 2A\sqrt{\zeta-1} + B \quad [B5]$$

The constants  $A$  and  $B$  are determined so that  $v(1, 0) = U_\infty \alpha$  and  $v(\infty) = 0$ . Then

$$v = U_\infty \alpha \left[ 1 - \sqrt{\frac{\zeta-1}{\zeta}} \right] \quad [B6]$$

Thus

$$\begin{aligned} v(\bar{z}, 0) &= -U_\infty \alpha \left[ 1 - \sqrt{\frac{\bar{z}-1}{\bar{z}}} \right], & \bar{z} < 0 \\ v(\bar{z}, 0) &= -U_\infty \alpha & 0 < \bar{z} < 1 \\ v(\bar{z}, 0) &= -U_\infty \alpha \left[ 1 - \sqrt{\frac{\bar{z}-1}{\bar{z}}} \right], & \bar{z} > 1 \end{aligned} \quad [B7]$$

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Substituting Equation [B7] in [B3],

$$y_e(\text{upper}; x) = \alpha \left[ -x + \frac{1}{2}(1 + 2\sqrt{x}) \sqrt{x + \sqrt{x}} + \frac{1}{4} \ln \left( 1 + 2\sqrt{x} - 2\sqrt{x + \sqrt{x}} \right) \right], x > 0$$

$$y_e(\text{lower}; x) = -\alpha x, \quad 0 < x < 1 \quad [\text{B8}]$$

$$y_e(\text{lower}; x) = \alpha \left[ -x - \frac{1}{2}(1 - 2\sqrt{x}) \sqrt{x - \sqrt{x}} - \frac{1}{4} \ln \left( -1 + 2\sqrt{x} + 2\sqrt{x - \sqrt{x}} \right) \right], x > 1$$

### EXACT THEORY

The exact wake shape in parametric form is easily found by making use of the results of the exact solution as presented in Reference 2, Chapter XII.

In the present notation and assuming a plate of unit chord,

$$x_e = -\frac{\cos \alpha}{2} \left[ \zeta_1 - \frac{\cos \alpha}{2} (\zeta_1^2 - 1) - 1 \right] + \frac{\sin^2 \alpha}{4} \left[ \zeta_1 \sqrt{\zeta_1^2 - 1} - \ln \left( \zeta_1 + \sqrt{\zeta_1^2 - 1} \right) \right], \quad 1 < \zeta_1 < -1 \quad [\text{B9}]$$

$$y_e(\text{upper}) = \frac{\sin \alpha}{2} \left[ \zeta_1 - \frac{\cos \alpha}{2} (\zeta_1^2 - 1) - 1 \right] + \frac{\sin \alpha \cos \alpha}{4} \left[ \zeta_1 \sqrt{\zeta_1^2 - 1} + \ln \left( \zeta_1 - \sqrt{\zeta_1^2 - 1} \right) \right], \quad 1 < \zeta_1 \quad [\text{B10}]$$

$$y_e(\text{lower}) = \frac{\sin \alpha}{2} \left[ \zeta_1 - \frac{\cos \alpha}{2} (\zeta_1^2 - 1) - 1 \right] - \frac{\sin \alpha \cos \alpha}{4} \left[ \zeta_1 \sqrt{\zeta_1^2 - 1} + \ln \left( -\zeta_1 + \sqrt{\zeta_1^2 - 1} \right) \right], \quad \zeta_1 < -1 \quad [\text{B11}]$$

where  $\zeta_1$  is a parameter.

Expanding Equations [B9], [B10], and [B11] in a power series in the angle of attack,

$$x_e = -\frac{1}{2} \left[ \zeta_1 - \frac{1}{2} \zeta_1^2 - \frac{1}{2} \right] + O(\alpha^2), \quad 1 < \zeta_1 < -1 \quad [\text{B12}]$$

$$y_e(\text{upper}) = \frac{\alpha}{2} \left[ \zeta_1 - \frac{1}{2} \zeta_1^2 - \frac{1}{2} \right] + \frac{\alpha}{4} \left[ \zeta_1 \sqrt{\zeta_1^2 - 1} - \ln \left( \zeta_1 - \sqrt{\zeta_1^2 - 1} \right) \right] + O(\alpha^3), \quad 1 < \zeta_1 \quad [\text{B13}]$$

$$y_e(\text{lower}) = \frac{\alpha}{2} \left[ \zeta_1 - \frac{1}{2} \zeta_1^2 - \frac{1}{2} \right] - \frac{\alpha}{4} \left[ \zeta_1 \sqrt{\zeta_1^2 - 1} + \ln \left( -\zeta_1 + \sqrt{\zeta_1^2 - 1} \right) \right] + O(\alpha^3), \quad \zeta_1 < -1 \quad [\text{B14}]$$

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Finally, upon elimination of the parameter  $\zeta_1$  from Equations (B12) - (B14), the following result is obtained

$$y_e(\text{exact}) = y_e(\text{linearized}) + O(\alpha^2) \quad \text{[B 15]}$$

where  $y_e(\text{linearized})$  is given in Equation (B 8).

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# APPENDIX C

## LISTING OF USEFUL THEORETICAL RESULTS

For convenience, the most useful theoretical results are listed below:

Relationship between hydrofoil shape and equivalent airfoil shape:

$$\frac{dy_0}{dx}(x) = \frac{d\bar{y}_0}{d\bar{x}}(\sqrt{x}) \quad [6]$$

Relationships between hydrofoil forces and moments, and equivalent airfoil forces and moments:

$$C_L = \bar{C}_M \quad [14]$$

$$C_M = \bar{C}_{M_3} \quad [15]$$

$$C_D = \frac{1}{8\pi} \bar{C}_L^2 \quad [16]$$

Forces, pitching moment about leading edge, and lift/drag ratio for a flat-plate hydrofoil ( $\sigma = 0$ ):

$$C_L = \frac{\pi\alpha}{2} \quad [17a]$$

$$C_M = \frac{5\pi\alpha}{32} \quad [18a]$$

$$C_D = \frac{\pi\alpha^2}{2} \quad [19a]$$

$$L/D = \frac{\pi}{2C_L} \quad [31]$$

Forces and pitching moment about leading edge for a flapped-plate hydrofoil ( $\sigma = 0$ ) (see Figure 7):

$$C_L = \frac{\epsilon}{2} [\pi - \theta_h + \sin \theta_h (2 - \cos \theta_h)] \quad [24]$$

$$C_M = \frac{\epsilon}{32} \left[ 5(\pi - \theta_h) + 14 \sin \theta_h - 7 \sin 2\theta_h + 2 \sin 3\theta_h - \frac{1}{4} \sin 4\theta_h \right] \quad [25]$$

$$C_D = \frac{\epsilon^2}{2\pi} [\pi - \theta_h + \sin \theta_h]^2 \quad [26]$$

where

$$\sqrt{\frac{x}{s}} = \frac{1}{2}(1 - \cos \theta_h) \quad [23]$$

Optimum hydrofoil lift/drag ratio ( $\sigma = 0$ ):

$$\left( \frac{L}{D} \right)_{opt} = \frac{8\pi}{C_L} \quad [21]$$

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Lift/drag ratio for relatively low-drag hydrofoils ( $\sigma = 0$ ) (see Figure 11):

$$\frac{L}{D} = \frac{\alpha' + \frac{2}{\pi} C_{L_d}}{\left(\alpha' + \frac{4}{5\pi} C_{L_d}\right)^2} \quad [42]$$

where  $C_{L_d}$  is the design lift coefficient and  $\alpha'$  is the difference between the operating angle of attack and the design angle of attack.

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12 Ship Design (Code 438)		38 Dr. J. H. Patterson, DT, Pasadena, Calif., via RSBAT, Los Angeles, Calif.
13-14 Power Res & Ship Proj (Code 439)	67 CO, Transportation Res & Dev Div, Fort Eustis, Va.	39 Dr. L. S. Stank, DR, R. Anthony Felt, Physics Lab, JSC of Mass, Cambridge, Mass., via WDCR, Div of Mass.
15 Power Storage (Code 477)		40 Dr. C. R. Soderberg, Div of Mech Engrs, MIT, Cambridge, Mass., via RSBAT, Boston
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17 Laboratory Design (Code 431)		42 Dr. L. T. Tilling, MIT, Cambridge, Mass., via RSBAT, Boston, Mass.
18 Submarine (Code 228)		43 Dr. W. L. Dumas, Mech Engrs Div, JSC, JSC Research Admin, Washington, D.C., via WDCR, Div of Mass.
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